Soluções dos exercícios de Análise Matemática IV

1. Equação diferencial de 1ª ordem

1.4 $x(t)=\frac{C}{t}+t,\ \ C\in\Re$

1.5 $x(t)=Ct^n+t^ne^t,\ \ C\in\Re$

1.6 $x(t)=e^{1/t^2}$

1.7 a) $x^3(t)=\frac{C\left(1+t^2\right)^{3/2}}{t^3}-1,\ \ C\in\Re$

b) $\frac{t^2}{2x^2}+\log|x|=K,\ \ K\in\Re\ \ \vee\ \ x(t)=0$

c) $\frac{t^2}{x^3}-\frac{1}{x}=C,\ \ C\in\Re$

1.8 $x(t)=\begin{cases}2\left(1-e^{-t}\right)\ \ se\ \ t\in[0,1]\\ 2e^{-t}(e-1)\ \ se\ \ t>1\end{cases}$

1.10 $x(t)=\frac{aC}{bC+e^{-at}},\ \ C\in\Re\ \ \vee\ \ x(t)=\frac{a}{b}$

1.14 $f(t)=-2\cos t+C,\ \ C\in\Re,\ \ \ x^2(t)\left(\frac{1}{2}C-\cos t\right)=K,\ \ \mathrm{K}\in\Re$

1.15 $a=1,\ \ \mathrm{e}^{-t}senx+t=C,\ \ C\in\Re$

1.16 $x(t)=C\cos t-\cos^2 t,\ \ C\in\Re$

1.17 a) $h(t,x)=\frac{3}{4}xt-\frac{1}{8}t^2+\varphi(x),\ \ \varphi(x)$ função arbitrária de x

b) $x^{3/2}t-\frac{t^2}{2}x^{1/2}+\int 2x^{-1/2}\varphi(x)dx=C,\ \ C\in\Re$ e $\varphi(x)$ função arbitrária de x

1.18 $t^3x+tx^3+2t+5x=9$

1.19 c) $2x^2+(2t-1)x+\frac{t^2}{2}+3t=C,\ \ C\in\Re$

1.20 a) $\mu(t)=e^{\int a(t)dt}$

b) $x(t)=Ce^{-t^2}-1,\ \ C\in\Re\ \ \lim_{t\to+\infty}x(t)=-1$

1.21 a) $x^2\left(Ce^{t^2}+t^2+1\right)=1,\ \ C\in\Re$

b) $x(t)=\dfrac{2}{1+2Ce^{t^2}},\ \ C\in\Re$

c) $x(t)=\dfrac{tgt+\sec t}{sent+C},\ \ C\in\Re$

d) $x(t)=\dfrac{1}{C\sqrt{|t^2-1|}-s},\ \ C\in\Re$

1.23 $x(t)=-e^{-t}$

1.24 $x(t)=C+\dfrac{t^2}{2},\ \ C\in\Re\ \ \vee\ \ x(t)=\left(\dfrac{t}{A}-\dfrac{1}{A^2}\right)e^{At}+B,\ \ A\neq 0\ \wedge\ B\in\Re$

1.26 $x(t)=3e^{t^2}-2$

1.27 a) $x_1(t)=1+t+\dfrac{t^3}{3},\ x_2(t)=1+t+t^2+\dfrac{2t^3}{3}+\dfrac{5t^4}{12}+\dfrac{2t^5}{15}+\dfrac{t^6}{18}+\dfrac{t^7}{63}$

b) *10*

c) $x(t)=\dfrac{e^{t^3/3}}{1-\int_0^t e^{l^3/3}dl}$

1.28 $x(t)=0\ \ \forall_{t\in\Re}\ ,\ \ \ \sqrt{x}=\dfrac{1}{2}\int_0^t g(s)ds$ são ambas soluções do PVI. Falha a condição de Lipschitz.

1.29 $x(t)\equiv 1,\ \ \ \int_1^x \dfrac{1}{\sqrt{s^2-1}}ds=t\ ,\ \ f\notin C^1([1,+\infty))$

1.30 $\theta\in\left(0,\dfrac{1}{T}\right)$

1.32 a) $x(t)=\dfrac{1}{2}\left(e^t+e^{-t}\right)=\cosh t$

b) $PVI\begin{cases} x''=x \\ x(0)=1 \\ x'(0)=0 \end{cases}$

2. Equação diferencial de ordem n

2.4

$a)\ x(t)=-2e^{2t}+3e^{t}$

$b)\ x(t)=(1-A)e^{-t}+(1-3t)Ae^{2t},\quad A\in\Re$

$c)\ x(t)=-te^{-t}+e^{-t}\log|t+1|+te^{-t}\log|t+1|+C_1e^{-t}+C_2te^{-t},\quad C_1,C_2\in\Re$

$d)\ x(t)=C_1e^{t}+C_2e^{-2t}-\frac{2}{5}\cos 2t-\frac{6}{5}sen2t,\quad C_1,C_2\in\Re$

$e)\ x(t)=C_1\cos 2t+C_2 sen2t+\frac{1}{4}tsen2t,\quad C_1,C_2\in\Re$

$f)\ x(t)=C_1\cos 3t+C_2 sen3t+e^{3t}\left(\frac{1}{18}t^2-\frac{1}{27}t+\frac{4}{81}\right),\quad C_1,C_2\in\Re$

$g)\ x(t)=e^{3t}\left(C_1+C_2t-\frac{2}{9}\cos 3t\right),\quad C_1,C_2\in\Re$

$h)\ x(t)=K_1e^{3t}+K_2e^{2t}+e^{t}\left(\frac{1}{2}t^2+\frac{3}{2}t+\frac{1}{4}\right),\quad K_1,K_2\in\Re$

$i)\ x(t)=K_1\cos t+K_2 sent+\cos t.\log|\cos t|+tsent,\quad K_1,K_2\in\Re$

2.5

$x(t)=C_1e^{-2t}+C_2te^{-2t}+C_3\cos t+C_4 sent$

$a)\ x(0)=C_3\quad x'(0)=C_4\quad x''(0)=-C_3\quad x'''(0)=-C_4$

$b)\ x(0)=C_1\quad x'(0)=-2C_1+C_2\quad x''(0)=4(C_1-C_2)\quad x'''(0)=-8C_1+12C_2$

$c)$ *impossível*

$d)\ x(0)=C_1+C_3\quad x'(0)=-2C_1+C_2+C_4\quad x''(0)=4C_1-4C_2-C_3\quad x'''(0)=-8C_1+12C_2-C_4$

2.6 $x(t)=C_1\cos t+C_2 sent,\quad C_1,C_2\in\Re$

2.7 b) $w(t)=At^{-3}+B\quad A\neq 0,\quad B\in\Re$

c) $x(t) = -4t^{-1} + t^3/4 \ \forall_{t>0}$

3. Sistemas de equações diferenciais

3.1 $\begin{cases} x_1(t) = C_1 e^t + C_2 t e^t \\ x_2(t) = C_2 e^t \end{cases} \qquad C_1, C_2 \in \Re$

3.2

3.3

a) $x'' - 6x' + 5x = 0$

b) $\begin{cases} x(t) = \frac{1}{4}\left(\mathrm{e}^{t}(5 - y_0) + \mathrm{e}^{5t}(y_0 - 1)\right) \\ y(t) = \frac{1}{4}\left(\mathrm{e}^{t}(5 - y_0) + 5\mathrm{e}^{5t}(y_0 - 1) - 4t\right) \end{cases}$

c) $y_0 = 1$

3.4 $\begin{cases} x_1(t) = C_1 \cos t - C_2 sent + 1 \\ x_2(t) = C_1 sent - C_2 \cos t + t \end{cases} \qquad C_1, C_2 \in \Re$

3.5 $a_{ii} < 0 \ \forall_{i=1,\ldots,n}$

3.6 $a = b = 1 \quad c = 3$

3.7 por exemplo $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ ou $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

3.8 $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

3.9

a) $\begin{cases} x_1(t) = e^t \\ x_2(t) = e^t(\cos t - sent) \\ x_3(t) = e^t(\cos t + sent) \end{cases}$

b) $\begin{cases} x_1(t) = e^{-t}\text{cost} \\ x_2(t) = e^{-t}(2\cos t + sent) \end{cases}$

3.10 $x^0 = \begin{bmatrix} x_1^{\ 0} \\ 0 \\ x_2^{\ 0} \end{bmatrix}$

3.11

$$\begin{cases} x_1(t) = 3e^{3t} \text{ - e}^{2\text{t}}(2 + \text{t}) \\ x_2(t) = e^{2t} \\ x_3(t) = 3e^{3t} \text{ - 2e}^{2\text{t}} \end{cases}$$

3.12 $\begin{cases} x_1(t) = \dfrac{1}{\cos t} \text{ - 1} + \text{sent.tgt - tsent} + \text{cost.log}|\text{cost}| \\ x_2(t) = \text{sent - tcost} - sent.\text{log}|\text{cost}| \\ x_3(t) = -\text{sent.tgt} + \text{tsent} - \text{cost.log}|\text{cost}| \end{cases}$

3.14 $e^{At} = I + A(e^t - 1)$

3.16 c) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \cos 2t + 5sen2t \\ 2\cos 2t - 3sen2t \end{bmatrix}$

3.17 $z(t) = \phi_1(t)\phi_1^{\ -1}(0)\phi_2(t)\phi_2^{\ -1}(0)z_0$

3.18

a) $\begin{cases} x_1(t) = e^{-2t}\text{cos3t} \\ x_2(t) = e^{-2t} sen3t \\ x_3(t) = e^{-4t} \end{cases}$

b) $\begin{cases} x_1(t) = te^t \\ x_2(t) = e^t - te^t \\ x_3(t) = e^t \end{cases}$

c) $\begin{cases} x_1(t) = e^t - te^t + t^2e^t \\ x_2(t) = e^t - te^t \\ x_3(t) = e^t \end{cases}$

3.19 b) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} e^t \\ 2e^t - \frac{9}{4}e^{-4t} + \frac{1}{4} \end{bmatrix}$

3.20 $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{3}e^t + \frac{1}{3}e^{4t} \\ \frac{5}{4}e^{2t} - \frac{1}{4}e^{-2t} \end{bmatrix}$

3.22 estável: a) e),   instável: b),   assintoticamente estável: c) d)

3.24 $\alpha \leq -\frac{3}{2}$

3.25 estável: c) d),   instável: a) b)

3.26 $x(t) = 1$ estável e $x(t) = 0$ instável

3.28

a) $(0,1)$ $(0,-1)$ $(1,0)$ instáveis; $(-1,0)$ estável

b) $(0, k\pi)_{k \in Z}$ instáves

c) $(k\pi, 0)_{k \in Z}$ instáveis

4. Equações com diferenças

4.1

a) $x_n = -\frac{3}{2}\left((-1)^n - 1\right)$

b) $x_n = 6.4^n + \frac{2}{3}\left(4^n - 1\right)$

c) $x_n = 1 - (-2)^n$

4.3

a) $x_n = C_1 \cos\frac{n\pi}{2} + C_2 sen\frac{n\pi}{2} + C_3 n \cos\frac{n\pi}{2} + C_4 n sen\frac{n\pi}{2} \quad C_i \in \Re, i = 1,2,3,4$

b) $x_n = C_1 + C_2 n - \frac{1}{2}n^2 + \frac{1}{6}n^3 \quad C_i \in \Re, i = 1,2$

c) $x_n = 2(-2)^n + 4(-3)^n - \frac{3}{2}n(-2)^n$

d) $y_t = C_1 2^t + C_2 3^t + 4 + \mathrm{t} \quad C_i \in \Re, i = 1,2$

e) $y_t = C_1 + C_2 t - \frac{t^2}{2} + \frac{t^3}{6} \quad C_i \in \Re, i = 1,2$

f) $y_t = C + 2^{t+1} + \frac{t}{6} + \frac{t^2}{2} + \frac{t^3}{3} \quad C \in \Re$, se $\Delta y_t = y_t - y_{t-1}$

$y_t = C + 2^t + \frac{t}{6} - \frac{t^2}{2} + \frac{t^3}{3} \quad C \in \Re$, se $\Delta y_t = y_{t+1} - y_t$

4.4 a) $\frac{1}{1-3x}$ b) $\frac{1}{1-x^2}$

4.5 $x_n = 5(-1)^n - 6(-2)^n$

5. Integração de funções complexas

5.1

a) $2\pi i$ b) $\frac{1}{(n-1)r^{n-1}}\left(1 - e^{-2\pi i(n-1)}\right), \quad n > 1$

5.2 $10 - \frac{8}{3}i$

5.3 $-\frac{15}{64}$

5.4 $-\frac{44}{3} - \frac{8}{3}i$

5.5 a) 0 b) $4\pi i$

5.7 a) 0 b) $2\pi i\left(e^2+2\right)$

5.8 a) $-2\pi i$ b) 0

5.9 a) $\dfrac{\pi i}{2}$ b) $2\pi i$

5.11 a) $\overline{D_1(0)}$ b) $D_2(i)$ c) C

5.12 a) $\dfrac{1}{2-z}$ b) $\dfrac{e(z-1)}{z-1-e}$

5.13 a) $\sum_{n\geq 0}\dfrac{e}{n!}(z-1)^n \quad z\in C$ b) $\sum_{n\geq 0}(-1)^n(z-1)^n \quad z\in D_1(1)$

5.14 a) $z=\dfrac{\pi}{2}+k\pi$ pólos simples $\forall k\in Z$

b) $z=0$ removível e $z=1$ pólo simples

c) $z=0$ pólo de ordem 2 e $z=k$ pólo simples $\forall k\in Z$

d) $z=\dfrac{\pi}{2}+k\pi$ pólos simples $\forall k\in Z$

e) $z=1$ removível e $z=\dfrac{-1\pm\sqrt{3}i}{2}$ pólos simples

f) $z=i$ removível e $z=1,-i$ pólos simples

g) $z=0$ essencial

5.15 $z=1$ essencial, $\sum_{n\geq 0}\dfrac{(-1)^n\pi^{2n+1}}{(2n+1)!}(z-1)^{-2n-1} \quad z\in A(1,0,+\infty)$

5.16 $e\sum_{n\geq 0}\dfrac{1}{n!}(z-1)^{-n} \quad z\in A(1,0,+\infty)$

5.17 $\operatorname{Res}\left(f, \frac{\pi}{2} + k\pi\right) = -1$

5.18 a) b) $\operatorname{Res}(f, 0) = 0$

5.19 a) b) 0 c) $-2\pi i$

5.20 $\pi i \left(\frac{e^{i}}{4} - \frac{e^{3i}}{6}\right)$

5.21 $z = 0$ essencial; $\operatorname{Res}(f, 0) = 0$; 0

5.22 a) $z = 0$ essencial, função analítica em $C \setminus \{0\}$

b) $\frac{\pi}{12}$